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1998 J. Phys. A: Math. Gen. 31 6029

(http://iopscience.iop.org/0305-4470/31/28/016)

J. Phys. A: Math. Gen. **31** (1998) 6029–6036. Printed in the UK PII: S0305-4470(98)90276-1

The *O*(*N*) nonlinear sigma model in the functional Schrödinger picture

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Received 5 January 1998

Abstract. We present a functional Schrödinger picture formalism of the $(1 + 1)$-dimensional $O(N)$ nonlinear sigma model. The energy density has been calculated to two-loop order using the wavefunctional of Gaussian form, and from which the non-perturbative mass gap of the boson fields has been obtained. The functional Schrödinger picture approach combined with the variational technique is shown to describe the characteristics of the ground state of the nonlinear sigma model in a transparent way.

1. Introduction

The ground state of interacting quantum fields, in general, has a complicated structure, making its investigation difficult: consequently attempts at using ordinary perturbation theories have met without much success. In particular, concerning spontaneous symmetry breaking and bound states, perturbative ground states lead to wrong results [1]. In this respect, the functional Schrödinger picture (FSP) approach with the variational approximation is expected to be a useful tool in examining the non-perturbative aspects of quantum field theory.

In contrast to the usual perturbative expansion, the Schrödinger picture approach has the merit that one does not need to specify a particular Fock basis for the ground state of the Hamiltonian under consideration. Therefore, where there is no well-defined Fock vacuum, this method appears to be a convenient choice [2].

In particular, the nonlinear sigma (NLS) model [3–6] has a non-trivial vacuum structure that is composed of particle–antiparticle pairs. The ground state is not easily tractable applying the usual perturbation expansion. Therefore, it appears as a natural candidate for application of the FSP approach. The aim of this paper is to analyse the NLS model in the framework of the FSP method combined with a variational approach [7–10], which is known to go beyond the perturbative scheme in some cases. We will show that the non-perturbative phenomena like the mass gap and asymptotic freedom can be described in the Schrödinger picture in a direct way.

The NLS model in lower dimensions has attracted much attention, since it has relevance to the low-energy limit of QCD as well as condensed-matter systems such as

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antiferromagnets. The NLS model in two dimensions is classically scale-invariant and asymptotically free [3–5]. According to Mermin–Wagner and Coleman [11], the continuous symmetry cannot be broken in $1+1$ dimensions; the massless Goldstone bosons tend to acquire their masses. It was also shown that there is a mapping between the NLS model and the effective long wavelength action of the quantum Heisenberg antiferromagnet [12].

In section 2, we briefly introduce the NLS model and its formulation in the FSP approach. In section 3, we first calculate the energy density to two-loop order using a Gaussian-type wavefunctional, and then derive the mass gap for the boson fields by minimizing the energy density. We will show that in the NLS model, the massive ground state is more stable than the massless one. In section 4, we give a brief summary and discussion of our results.

2. The nonlinear sigma model in the functional Schrödinger picture

We start with the $O(N)$-invariant Lagrangian density [3, 5]

$$\mathcal{L} = \frac{1}{2\lambda} \partial_\mu \Phi_a \partial^\mu \Phi_a \tag{1}$$

where N scalar fields $\Phi_a,\ a = 1, \ldots, N$, obey the constraint

$$\sum_{a=1}^{N} \Phi_a \Phi_a = 1. \tag{2}$$

This constraint makes the theory quite complicated, since the N components of the scalar field Φ are mutually dependent on each other. The coupling constant λ is a measure of the strength of the self interaction of the N scalar fields Φ_a, and a small value of λ corresponds to a weak interaction. The constraint in (2) means that one degree of freedom among the N variables, Φ_a, is not a real dynamical variable [13]. Thus, we follow the standard prescription to get rid of the Nth field Φ_N through the following nonlinear transformation [4]:

$$\Phi_a = \frac{\phi_a}{(1+\phi^2/4)} \qquad \Phi_N = \frac{(1-\phi^2/4)}{(1+\phi^2/4)} \tag{3}$$

where $\phi^2 \equiv \sum_{a=1}^{N-1} {\phi_a}^2$. Substituting these expressions in the Lagrangian, we find the equivalent Lagrangian involving the $N-1$ fields ϕ_a:

$$\mathcal{L} = \frac{1}{2\lambda} \frac{\partial_\mu \phi_a \partial^\mu \phi_a}{(1+\phi^2/4)^2}. \tag{4}$$

Here, and in what follows, summation over the repeated indices is implied; otherwise, a comment will be given explicitly.

The NLS model in terms of the ϕ_a has no mass parameter, so these fields are classically massless Goldstone bosons. However, the Goldstone bosons originate from the breakdown of the continuous $O(N)$ symmetry, which cannot occur in this case [11]. In what follows, it will be seen that the Goldstone bosons become massive through the quantum mechanical self-interaction.

The canonical quantization procedure of the classical system requires the conjugate momentum of the field ϕ_a, which becomes

$$\begin{aligned} \pi_a &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}_a} \\ &= \frac{1}{\lambda} \frac{\dot{\phi}_a}{(1+\phi^2/4)^2}. \end{aligned} \tag{5}$$

Thus, the Hamiltonian is written in terms of the canonical variables, π_a and ϕ_a as

$$H = \pi_a \dot{\phi}_a - \mathcal{L}(\phi_a, \dot{\phi}_a)$$
$$= \frac{1}{2}\lambda\left(1 + \frac{\phi^2}{4}\right)^2 \pi^2 + \frac{1}{2\lambda}\frac{\phi'^2}{(1+\phi^2/4)^2} \tag{6}$$

where we have used the conventions $\phi^2 \equiv \sum_{a=1}^{N-1}\phi_a^2$, $\pi^2 \equiv \sum_{a=1}^{N-1}\pi_a^2$ and $\phi'^2 \equiv \sum_{a=1}^{N-1}(\nabla\phi_a \cdot \nabla\phi_a)$.

In the quantum theory, the dynamical field variable $\phi_a(x)$ and $\pi_a(x)$ become field operators. These π_a satisfy the canonical equal-time commutation relations with the ϕ_a such that

$$[\phi_a(x), \pi_b(y)]_{x_0=y_o} = \mathrm{i}\delta_{ab}\delta(x-y). \tag{7}$$

In the FSP representation of its quantum theory, the scalar field operator $\phi_a(x)$ and its conjugate momentum $\pi_a(x)$ are realized [7–9] as

$$\phi_a(x) \to \phi_a(x)$$
$$\pi_a(x) \to -\mathrm{i}\hbar\frac{\delta}{\delta\phi_a(x)}. \tag{8}$$

We now employ a Gaussian-type wavefunctional [9] with two variational function parameters $G_{ab}(x,y)$ and $\hat{\phi}_a(x)$:

$$\Psi[\phi] = \frac{1}{[\det(2\pi\hbar G)]^{1/4}}\exp\left[-\int_{x,y}(\phi_a(x) - \hat{\phi}_a(x))\frac{G_{ab}^{-1}(x,y)}{4\hbar}(\phi_b(y) - \hat{\phi}_b(y))\right] \tag{9}$$

where repeated indices over a or b mean sums; an explicit comment will be given otherwise. This wavefunctional may be seen to obey the constraints

$$\langle\Psi|\Psi\rangle = 1$$
$$\langle\Psi|\phi_a(x)|\Psi\rangle = \hat{\phi}_a(x) \tag{10}$$
$$\langle\Psi|\pi_a(x)|\Psi\rangle = 0.$$

Here, $\hat{\phi}_a(x)$ is the expectation value of the field operator $\phi_a(x)$, and the expectation value of the momentum operator has been chosen to be zero. From the above trial wavefunctional, we readily obtain the following results, which are needed for the calculation of the expectation value of the Hamiltonian:

$$\langle\phi_a(x)\phi_b(y)\rangle = \hat{\phi}_a(x)\hat{\phi}_b(y) + \hbar G_{ab}(x,y)$$
$$\langle\pi_a(x)\pi_b(y)\rangle = \tfrac{1}{4}\hbar G_{ab}^{-1}(x,y). \tag{11}$$

3. Large-N calculations

In order to obtain the expectation value of the Hamiltonian with the Gaussian trial wavefunctional $\langle\Psi|H|\Psi\rangle$, it is necessary to handle the field operator in $(1+\phi^2/4)^{-2}$ properly. For that purpose, we expand it in terms of $\phi^2/4$, which involves an infinite number of terms of even powers of $\phi_a(x)$ fields.

Thus, we have to evaluate terms as follows:

$$\langle\Psi|\overbrace{\phi^2(x)\cdots\phi^2(x)}\pi^2(x)|\Psi\rangle \qquad \text{and} \qquad \langle\Psi|\overbrace{\phi^2(x)\cdots\phi^2(x)}\phi'^2(x)|\Psi\rangle \tag{12}$$

which cannot be calculated in closed forms, except for some special limiting cases. Due to this inability, we require an approximation scheme. Here, we will resort to large-N calculations [14]. The following example will show why the large-N approximation is useful in the present problem:

$$\langle\Psi|\phi^2(x)\phi^2(x)\pi^2(x)|\Psi\rangle = \left[(\hat{\phi}_a\hat{\phi}_a + \overline{h}G_{aa})(\hat{\phi}_b\hat{\phi}_b + \overline{h}G_{bb})\tfrac{1}{4}\overline{h}G^{-1}_{cc}\right]$$
$$+\overline{h}^2\left[\hat{\phi}_a G_{ab}\hat{\phi}_b G^{-1}_{cc} - 1\hat{\phi}_a\hat{\phi}_a\delta_{bb}\delta(0) + \tfrac{1}{2}\overline{h}G_{ab}G_{ba}G^{-1}_{cc} - \overline{h}G_{aa}\delta_{bb}\delta(0)\right]$$
$$-2\overline{h}^2\left[\hat{\phi}_a\hat{\phi}_a + \overline{h}G_{aa}\right] \tag{13}$$

where the three terms in the brackets on the right-hand side are of the order of N^3, N^2 and N respectively. Thus in the large-N limit, only the leading terms of N^3 order dominate:

$$\langle\Psi|\phi^2(x)\phi^2(x)\pi^2(x)|\Psi\rangle \xrightarrow{\text{Large } N} (\hat{\phi}_a\hat{\phi}_a + \overline{h}G_{aa})(\hat{\phi}_b\hat{\phi}_b + \overline{h}G_{bb})\tfrac{1}{4}\overline{h}G^{-1}_{cc}$$
$$= \langle\Psi|\phi^2(x)|\Psi\rangle\langle\Psi|\phi^2(x)|\Psi\rangle\langle\Psi|\pi^2(x)|\Psi\rangle. \tag{14}$$

Therefore it is clear that in the large-N limit, the expectation values of the composite operators [4] become

$$\langle\Psi|\phi^2(x)\cdots\phi^2(x)\pi^2(x)(\text{or } \phi'^2(x))|\Psi\rangle$$
$$= \langle\Psi|\phi^2(x)|\Psi\rangle\cdots\langle\Psi|\phi^2(x)|\Psi\rangle\langle\Psi|\pi^2(x)(\text{or } \phi'^2(x))|\Psi\rangle. \tag{15}$$

To construct an $1/N$ expansion [14] in a systematic way, we define a new parameter g such that

$$g \equiv \lambda N \tag{16}$$

where g is constrained to be finite. Thus, we are allowed to write the Hamiltonian expectation value in the following form:

$$\langle\Psi|H|\Psi\rangle = \frac{1}{2}\frac{g}{N}\left[1 + \frac{\langle\phi^2\rangle}{4}\right]^2 \langle\pi^2\rangle + \frac{1}{2}\frac{N}{g}\left[1 + \frac{\langle\phi^2\rangle}{4}\right]^{-2}\langle\phi'^2\rangle \tag{17}$$

where on the right-hand side, the expectation value has been taken with respect to the Gaussian wavefunctional in (9). Using the results for $\langle\phi^2\rangle$, $\langle\pi^2\rangle$ in (11), this equation can be rewritten as

$$\langle H\rangle = \frac{1}{2}\frac{g}{N}\left[1 + \frac{1}{4}(\hat{\phi}_a\hat{\phi}_a + \overline{h}G_{aa})\right]^2\frac{\overline{h}}{4}G^{-1}_{cc}$$
$$+\frac{1}{2}\frac{N}{g}\left[1 + \frac{1}{4}(\hat{\phi}_a\hat{\phi}_a + \overline{h}G_{aa})\right]^{-2}(\nabla\hat{\phi}_c\nabla\hat{\phi}_c - \nabla^2\overline{h}G_{cc}). \tag{18}$$

At this stage, we confine ourselves to the constant field configuration for the $\hat{\phi}_a(x)$ fields, so that the square of the gradient of $\hat{\phi}_a(x)$ in (18) vanishes. The scheme to expand $\langle\Psi|H|\Psi\rangle$ in powers of $\overline{h}$ and discard terms higher than second order in $\overline{h}^2$ will be adopted in the below. The $\overline{h}$ expansion is equivalent to the loop expansion [5, 15]. Thus, we are going to study the system to two-loop order. The energy density given in (18) can be expanded to second order in $\overline{h}$ to yield

$$\langle H\rangle = \frac{1}{2}\frac{g}{N}f^2(\hat{\phi})\left[1 + \frac{1}{2}\frac{\overline{h}G_{aa}}{f(\hat{\phi})}\right]\frac{\overline{h}}{4}G^{-1}_{cc} - \frac{1}{2}\frac{N}{g}f^{-2}(\hat{\phi})\left[1 - \frac{1}{2}\frac{\overline{h}G_{aa}}{f(\hat{\phi})}\right](\nabla^2\overline{h}G_{cc}) \tag{19}$$

where we have used the definition $f(\hat{\phi}) = 1 + \frac{1}{4}\hat{\phi}^2$.

We now vary this equation with respect to $G_{bb}(x, x)$ to determine the parameters $G_{bb}(x, x)$ which minimize the energy expectation value. Thus, one obtains the following relation:

$$\delta\langle H\rangle = \frac{g}{N}\frac{\hbar f(\hat{\phi})}{8}\left[\frac{\hbar}{2}G_{aa}^{-1} - f(\hat{\phi})\left[1 + \frac{\hbar}{2f(\hat{\phi})}G_{aa}\right]^2\frac{1}{G_{bb}{}^2}\right]\delta G_{bb}$$
$$+\frac{N}{g}\frac{\hbar}{2f^2(\hat{\phi})}\left[-\left[1 - \frac{\hbar}{2f(\hat{\phi})}G_{aa}\right]\nabla^2 + \frac{\hbar}{2f(\hat{\phi})}\nabla^2 G_{aa}\right]\delta G_{bb} = 0. \tag{20}$$

Here and below, we will use the following notation. Repeated indices over the letter b do not indicate a sum, whereas repeated indices over a do mean a sum. This equation gives the relation that the variational parameter G_{bb} must satisfy:

$$G_{bb}^{-2}(x, y) = \left[-\frac{N^2}{g^2}\frac{4}{f^4(\hat{\phi})}\left(1 - \frac{3}{2}\frac{\hbar G_{aa}(z, z)}{f(\hat{\phi})}\right)\nabla_x^2 + \frac{1}{2}\frac{\hbar}{f(\hat{\phi})}G_{aa}^{-1}(z, z)\right]\delta(x - y) \tag{21}$$

which has also been expanded in powers of $\hbar$, and terms higher than $\hbar$ have been discarded, so that only terms up to $\hbar^2$ can be retained in (19). Since it is practically impossible to solve this equation directly, we separate the equation into two parts as follows:

$$G_{bb}(x, y) = \frac{g}{N}\frac{f^2(\hat{\phi})}{2}\frac{1}{[1 - (3\hbar/2f)G_{aa}(z, z)]^{1/2}}\int\frac{\mathrm{d}^n p}{(2\pi)^n}\frac{1}{\sqrt{p^2 + m^2}}\exp[\mathrm{i}p(x - y)] \tag{22}$$

and

$$m^2 = \frac{g^2}{N^2}\frac{\hbar f^3(\hat{\phi})}{8}G_{aa}^{-1}(x, x) \tag{23}$$

where the latter turns out to define the mass parameter of the boson operator $\phi(x)$, as will be seen in (26).

We analyse the equation using the following iterative method. First, we approximate the unknown function $G_{bb}(x, y)$ by $G_{bb}^{(0)}(x, y)$:

$$G_{bb}^{(0)}(x, y) = \frac{g}{N}\frac{f^2(\hat{\phi})}{2}\int\frac{\mathrm{d}^n p}{(2\pi)^n}\frac{1}{\sqrt{p^2 + m^2}}\exp[\mathrm{i}p(x - y)]. \tag{24}$$

Second, to improve this approximation, we substitute the equation back in the coefficient of the right-hand side of (22). Thus, we have

$$G_{bb}^{(1)}(x, y) = \frac{g}{N}\frac{f^2(\hat{\phi})}{2}\frac{1}{[1 - (3\hbar/2f)G_{aa}^{(0)}(x, x)]^{1/2}}\int\frac{\mathrm{d}^n p}{(2\pi)^n}\frac{1}{\sqrt{p^2 + m^2}}\exp[\mathrm{i}p(x - y)]. \tag{25}$$

Note that the multiplicative factor in front of the integral has a divergence involving a cutoff Λ. This must be removed by a proper renormalization of the wavefunction $\phi(x)$. Then the wavefunction renormalized expression for $G_{bb}(x, y)$ becomes

$$G_{bb}^{(f)}(x, y) = \frac{g}{N}\frac{f^2(\hat{\phi})}{2}\int\frac{\mathrm{d}^n p}{(2\pi)^n}\frac{1}{\sqrt{p^2 + m^2}}\exp[\mathrm{i}p(x - y)]. \tag{26}$$

This form will be used in the subsequent discussions in calculating the energy expectation value. As a result of this renormalization, equation (21) is now rewritten as

$$G_{bb}^{-2}(x, y) = \left[-\frac{N^2}{g^2}\frac{4}{f^4(\hat{\phi})}\nabla_x^2 + \frac{1}{2}\frac{\hbar}{f(\hat{\phi})}G_{aa}^{-1}(z, z)\right]\delta(x - y). \tag{27}$$

Let us now evaluate the Hamiltonian expectation value using equations (23) and (26). The $\nabla^2 G_{aa}$ in the Hamiltonian can be calculated multiplying (27) by $G_{bb}(y, z)$ and integrating over the volume $\int \mathrm{d}y$:

$$\nabla^2 G_{aa}(x,x) = -\frac{g^2}{N^2}\frac{f^4(\hat{\phi})}{4}G_{aa}^{-1}(x,x)\left[1 - \frac{1}{2}\frac{\hbar}{f(\hat{\phi})}G_{aa}(x,x)\right]. \tag{28}$$

Since the classical Lagrangian density in (4) has no potential energy part, we are allowed to set $\hat{\phi}_a = 0$, which leads to

$$f(\hat{\phi}) = 1. \tag{29}$$

We will consider only the $(1+1)$-dimensional case, so n in (26) is chosen to be 1. Thus, the Hamiltonian to second order in $\hbar$, i.e. to two-loop order, is given by

$$\begin{aligned}\langle\Psi|H|\Psi\rangle &= \frac{g}{N}\frac{\hbar}{8}G_{aa}^{-1}(x,x)\left[1 + \frac{\hbar}{2}G_{aa}(x,x)\right] + \frac{g}{N}\frac{\hbar}{8}G_{aa}^{-1}(x,x)\left[1 - \frac{\hbar}{2}G_{aa}(x,x)\right]^2 \\ &= \frac{g}{N}\frac{\hbar}{4}G_{aa}^{-1}(x,x)\left[1 - \frac{\hbar}{4}G_{aa}(x,x)\right]. \end{aligned}\tag{30}$$

Using the mass defining relation (23) and two-point Green function (26), one can evaluate $\langle H\rangle$:

$$\begin{aligned}\frac{\langle H\rangle}{N} &= \frac{2m^2}{g}\left(1 - \frac{\hbar}{4}G_{aa}(x,x)\right) \\ &= \frac{2m^2}{g}\left[1 - \frac{\hbar g}{8}\int_{-\Lambda}^{\Lambda}\frac{\mathrm{d}p}{2\pi}\frac{1}{\sqrt{p^2+m^2}}\right] \\ &= \frac{2m^2}{g}\left[1 - \frac{\hbar g}{16\pi}\ln\left(\frac{4\Lambda^2}{m^2}\right)\right]\end{aligned}\tag{31}$$

where Λ is an ultraviolet momentum cutoff. To make the energy density finite, we may renormalize the parameter of the theory. Defining the renormalized coupling constant g_r

$$\frac{1}{g} = \frac{1}{g_r} + \frac{\hbar}{16\pi}\ln\left(\frac{4\Lambda^2}{\mu^2}\right) \tag{32}$$

we obtain the finite-energy density with the renormalization scale μ:

$$\frac{\langle H\rangle}{N} = \frac{2m^2}{g_r} + \frac{\hbar m^2}{8\pi}\ln\left(\frac{m^2}{\mu^2}\right). \tag{33}$$

Note that this is the result up to $\hbar^2$, since the mass parameter m^2 is of order $\hbar$. This energy expectation value has a minimum away from the origin, since it is concave upward as m^2 increases.

Hence, by minimizing the energy expectation value with respect to m^2, one can derive the mass gap

$$\begin{aligned}\frac{\partial\langle H\rangle/N}{\partial m^2} &= \frac{2}{g_r} + \frac{\hbar}{8\pi}\ln\left(\frac{em^2}{\mu^2}\right) \\ &= 0.\end{aligned}\tag{34}$$

From this relation, one obtains the dynamically generated mass gap:

$$\langle m^2\rangle = \mu^2\exp\left[-1 - \frac{16\pi}{\hbar g_r}\right]. \tag{35}$$

One may note that this result for the mass gap m is actually the same as those obtained through other methods in the literature [3–5]. Remembering the invariance of the system under renormalization group, the mass gap can be rewritten as a function of μ alone in an equivalent way, as in [6].

The NLS model has no dimensional parameters; the coupling g is dimensionless in two dimensions. However, we arrived at a dimensional parameter m^2; this phenomenon is an example of dimensional transmutation. At this value of the mass gap, the energy density to two-loop order becomes

$$\begin{aligned}\frac{\langle H\rangle}{N} &= \frac{2\langle m^2\rangle}{g_r} + \frac{\hbar\langle m^2\rangle}{8\pi}\ln\left(\frac{\langle m^2\rangle}{\mu^2}\right)\\ &= -\frac{\hbar}{8\pi}\mu^2\exp\left[-1-\frac{16\pi}{\hbar g_r}\right].\end{aligned} \tag{36}$$

Note that the negative sign of the energy density indicates that the massive ground state is more stable than the massless one in the NLS model.

We now return to the mass-defining equation, equation (23). One can see that equation (27) leads to

$$G^{-1}_{aa}(x,x) = \frac{2N}{g}\int_{-\infty}^{+\infty}\frac{\mathrm{d}p}{2\pi}\sqrt{p^2+m^2}. \tag{37}$$

Thus, differentiating equation (23) with respect to m^2, one finds the relation between the coupling constant g and the cutoff Λ:

$$\begin{aligned}\frac{1}{g} &= \frac{\hbar}{8}\int_{-\infty}^{\infty}\frac{\mathrm{d}p}{2\pi}\frac{1}{\sqrt{p^2+m^2}}\\ &= \frac{\hbar}{16\pi}\ln\left(\frac{4\Lambda^2}{m^2}\right).\end{aligned} \tag{38}$$

This relation shows that as Λ becomes large, the coupling constant g approaches zero, thus satisfying asymptotic freedom. This equation can be also rewritten in terms of the renormalized coupling g_r and the renormalization mass scale μ in (32).

4. Conclusion

In this paper, the functional Schrödinger picture approach has been applied to the analysis the $O(N)$ nonlinear sigma (NLS) model. We have considered the $O(N)$ NLS model in the large-N limit and calculated the energy expectation value to second order in $\hbar$ (two-loop order) systematically, using a wavefunctional of Gaussian form. The Schrödinger picture approach combined with the variational technique produced the mass gap and the asymptotic freedom of the ground state for the $O(N)$ NLS model in a straightforward manner.

Most of the literature on the $O(N)$ NLS model adopts the Lagrangian formalism to investigate its non-perturbative phenomena, where a composite auxiliarly field $\sigma(x) = \sum_a \Phi_a(x)\Phi_a(x)$ is usually introduced. However, here we discuss its non-perturbative phenomena directly without resorting to the superfluous auxiliarly field.

The extension of our calculations to three dimensions will be straightforward; the difference will be to write the Green function in (26) in the corresponding three-dimensional form and carry out the integral in three dimensions.

Acknowledgments

We thank Professor K-S Soh and J H Yee for helpful discussions. This work was supported in part by the Korea Ministry of Education (BSRL-96-2425), the Korea Science and Engineering Foundation through Project 95-0701-04-01-03 and through the SRC Program of SNU-CTP. DKK also acknowledges the Korea Research Foundation for support through the domestic postdoctorial program.

References

[1] Bardeen J, Cooper L N and Schrieffer J R 1957 *Phys. Rev.* **108** 1175
Nambu Y and Jona-Lasinio G 1961 *Phys. Rev.* **122** 345
[2] Floreanini R and Jackiw R 1988 *Phys. Rev.* D **37** 2206
[3] Brézin E and Zinn-Justin J 1976 *Phys. Rev.* D **14** 2615
Novikov V A, Shifman M A, Vainshtein A I and Zakharov V I 1984 *Phys. Rep.* **116** 103
Davis A C, Gracey J A, Macfarlane A J and Mitchard M G 1989 *Nucl. Phys.* B **314** 439
[4] Bardeen W A, Lee B W and Shrock R E 1976 *Phys. Rev.* D **14** 985
del R Gaztelurrutia T and Davis A C 1990 *Nucl. Phys.* B **347** 319
[5] Zinn-Justin J 1989 *Quantum Field Theory and Critical Phenomena* (Oxford: Oxford University Press)
[6] Hasenfratz P, Maggiore M and Niedermayer F 1990 *Phys. Lett.* **245B** 522
Hasenfratz P and Niedermayer F 1990 *Phys. Lett.* **245B** 529
[7] Cornwall J M, Jackiw R and Tomboulis E 1974 *Phys. Rev.* D **10** 2428
Jackiw R and Kerman A 1979 *Phys. Lett.* **71A** 158
Duncan A, Meyer-Ortmanns H and Roskies R 1987 *Phys. Rev.* D **36** 3788
[8] Kim S K, Yang J, Soh K S and Yee J H 1989 *Phys. Rev.* D **40** 2647
Kim S K, Soh K S and Yee J H 1990 *Phys. Rev.* D **41** 1345
Amelino-Camelia G and Pi S-Y 1993 *Phys. Rev.* D **47** 2356
Dunne G 1994 *Phys. Rev.* D **50** 5321
[9] Pi S-Y and Samiullah M 1987 *Phys. Rev.* D **36** 3128
[10] Noh H S, Kim C K and Nahm K 1995 *Phys. Lett.* **204A** 162
Noh H S, Kim C K and Nahm K 1996 *Phys. Lett.* **210A** 317
Song J, Hyun S and Kim C K 1996 *J. Korean Phys. Soc.* **29** 821
[11] Mermin N D and Wagner H 1966 *Phys. Rev. Lett.* **17** 1133
Coleman S 1973 *Commun. Math. Phys.* **31** 259
[12] Haldane F D M 1983 *Phys. Lett.* **93A** 464
Haldane F D M 1983 *Phys. Rev. Lett.* **50** 1153
[13] Dirac P A M 1964 *Lectures on Quantum Mechanics* (New York: Yeshiva University)
Henneaux M and Teitelboim C 1992 *Quantization of Gauge Systems* (Princeton, NJ: Princeton University Press)
[14] Coleman S 1985 *Aspects of Symmetry* (Singapore: World Scientific/Cambridge: Cambridge University Press)
[15] Nambu Y 1966 *Phys. Lett.* **26B** 626